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HW7 , Math 531, Spring 2014

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QUESTION 1. (i) Let R be a commutative ring with $1 \neq 0$ and I be a proper ideal of R. Then we know that $I[X]$ is a proper ideal of $R[X]$. Prove that $\sqrt{I[X]} = \sqrt{I}[X]$.

(ii) Let $f : R \to S$ be a ring epimorphism, where R and S are commutative rings with one. Given r is a prime element of R. Give an example of such f, where $f(r)$ is not a prime element of S, $f(r)$ is not a unit of S, and $f(r) \neq 0$.

(iii) Let $f : R \to S$ be a ring monomorphism, where R and S are commutative rings with one. Given r is an irreducible element of R. Prove that $f(r)$ is an irreducible element of $Image(f)$. Give an example of such f, where $f(r)$ is not an irreducible element of S.

(iv) Let R be a commutative ring with $1 \neq 0$. Give me an example of a proper ideal D of $R[X]$ such that $D \neq I[X]$ for some proper ideal I of R.

(v) Let R and S be commutative rings with $1 \neq 0$, and $D = R \times S$. Let J be an ideal of D. It is easy to see that $J = I_1 \times I_2$ for some ideal I_1 of R, and some ideal I_2 of S (you don't need to show that).

a. Prove that $\frac{D}{I_1 \times I_2}$ is ring-isomorphic to $R/I_1 \times S/I_2$.

b. Let L be a proper ideal of D. Prove that L is prime if and only if $L = P \times S$ for some prime ideal P of R or $L = R \times K$ for some prime ideal K of S.

c. Let L be a proper ideal of D. Prove that L is maximal if and only if $L = P \times S$ for some maximal ideal P of R or $L = R \times K$ for some maximal ideal K of S.

QUESTION 2. Note that every ideal of a ring R is a subring of R. Let R be a commutative ring with $1 \neq 0$.

(i) Let J, I be ideals of R. Prove that $\frac{J+I}{I}$ is ring isomorphic to $\frac{J}{J \cap I}$.

(ii) Let M, K be distinct maximal ideals of R. Prove that $\frac{M}{M \cap K}$ is a field

QUESTION 3. Let R and S are commutative rings with one.

(i) Let $I \subseteq J$ be proper ideals of R. Prove that $\frac{J}{I}$ is a prime ideal of R/I if and only if J is a prime ideal of R .

(ii) Let f be a ring epimorphism from R onto S, and $ker(f) \subseteq J$ be proper ideals of R. Prove that $f(J)$ is a prime ideal of S if and only if J is a prime ideal of R.

(iii) Let f be a ring epimorphism from R onto S. Let D be a prime ideal of S. Prove that $D = f(L)$ for some prime ideal L of R such that $Ker(f) \subseteq L$.

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